An Overview of Evolutionary Multi-Objective Optimization

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Outline of Topics

1. Introduction
    - A Taxonomy of MOEAs
    - Number of papers on EMOO
2. Some Applications
3. Current State and Future Challenges

Motivation

Most problems in nature have several (possibly conflicting) objectives to be satisfied (e.g., design a bridge for which want to minimize its weight and cost while maximizing its safety). Many of these problems are frequently treated as single-objective optimization problems by transforming all but one objective into constraints.

Formal Definition

Find the vector $\vec{x}^* = \left[x_1^*, x_2^*, \ldots, x_n^*\right]^T$ which will satisfy the m inequality constraints:

$$g_i(\vec{x}) \leq 0 \quad i = 1, 2, \ldots, m \tag{1}$$

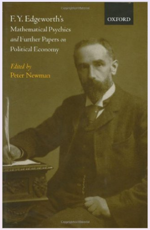
the p equality constraints

$$h_i(\vec{x}) = 0 \quad i = 1, 2, \ldots, p \tag{2}$$

and will optimize the vector function

$$\vec{f}(\vec{x}) = [f_1(\vec{x}), f_2(\vec{x}), \ldots, f_k(\vec{x})]^T \tag{3}$$

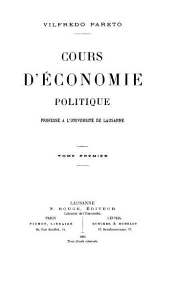
Notion of Optimality in MOPs



Having several objective functions, the notion of "optimum" changes, because in MOPs, the aim is to find good compromises (or "trade-offs") rather than a single solution as in global optimization.

The notion of "optimum" that is most commonly adopted is that originally proposed by Francis Ysidro Edgeworth (in 1881) in his book entitled **Mathematical Psychics**.

Notion of Optimality in MOPs



VILFREDO PARETO

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1896
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This notion was generalized by the italian economist Vilfredo Pareto (in 1896) in his book **Cours d'Economie Politique**. Although some authors call *Edgeworth-Pareto optimum* to this notion (originally called **ophelimity**) it is normally preferred to use the most commonly accepted term: **Pareto optimum**.

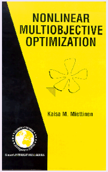
Notion of Optimality in MOPs

Pareto Optimality

We say that a vector of decision variables $\vec{x}^* \in \mathcal{F}$ is **Pareto optimal** if there does not exist another $\vec{x} \in \mathcal{F}$ such that $f_i(\vec{x}) \leq f_i(\vec{x}^*)$ for all $i = 1, \ldots, k$ and $f_j(\vec{x}) < f_j(\vec{x}^*)$ for at least one j (assuming that all the objectives are being minimized).

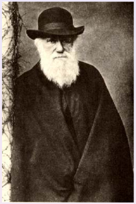
In words, this definition says that $\vec{x}^*$ is **Pareto optimal** if there exists no feasible vector of decision variables $\vec{x} \in \mathcal{F}$ which would decrease some criterion without causing a simultaneous increase in at least one other criterion. This concept normally produces a set of solutions called the **Pareto optimal set**. The vectors $\vec{x}^*$ corresponding to the solutions included in the Pareto optimal set are called **nondominated**. The image of the Pareto optimal set is called the **Pareto front**.

Mathematical Programming Techniques



Currently, there are over 30 mathematical programming techniques for multiobjective optimization. However, these techniques tend to generate elements of the Pareto optimal set one at a time. Additionally, most of them are very sensitive to the shape of the Pareto front (e.g., they do not work when the Pareto front is concave or when the front is disconnected).

Evolutionary Algorithms

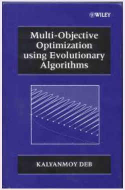


The idea of using techniques based on the emulation of the mechanism of natural selection to solve problems can be traced as long back as the 1930s. However, it was not until the 1960s that the three main techniques based on this notion were developed: genetic algorithms [Holland, 1962], evolution strategies [Schwefel, 1965] and evolutionary programming [Fogel, 1966]. These approaches are now collectively denominated **evolutionary algorithms**.

Evolutionary Algorithms

Evolutionary algorithms seem particularly suitable to solve multiobjective optimization problems, because they deal simultaneously with a set of possible solutions (the so-called population). This allows us to find several members of the Pareto optimal set in a single run of the algorithm, instead of having to perform a series of separate runs as in the case of the traditional mathematical programming techniques. Additionally, evolutionary algorithms are less susceptible to the shape or continuity of the Pareto front (e.g., they can easily deal with discontinuous or concave Pareto fronts), whereas these two issues are a real concern for mathematical programming techniques.

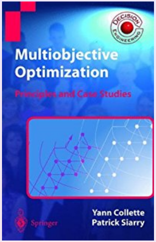
A Taxonomy of MOEAs



The Old Days (1980s to early 1990s)

- Non-Elitist Non-Pareto-based Methods
  - Lexicographic Ordering
  - Linear Aggregating Functions
  - VEGA
  - ε-Constraint Method
  - Target Vector Approaches

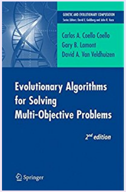
A Taxonomy of MOEAs



The Old Days (early 1990s to mid 1990s)

- Non-Elitist Pareto-based Methods
  - Pure Pareto ranking
  - MOGA
  - NSGA
  - NPGA and NPGA 2

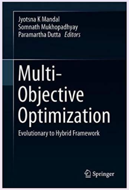
A Taxonomy of MOEAs



Contemporary Approaches (late 1990s to early 2000s)

- Elitist Pareto-based Methods
  - SPEA and SPEA2
  - NSGA-II
  - PAES, PESA and PESA II
  - Micro-genetic Algorithm for Multi-Objective Optimization and μGA^2
  - Many others (most of them already forgotten...)

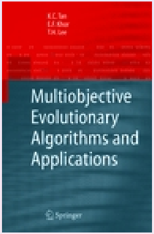
A Taxonomy of MOEAs



Recent Approaches (from early 2000s until today)

- MOEA/D (and its many variants)
- Indicator-Based Approaches
  - SMS-EMOA
  - HyPE
  - Other Approaches
- NSGA-III (and its many variants)

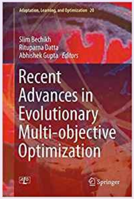
A Taxonomy of MOEAs



In general, modern MOEAs consist of two basic components:

- A selection mechanism that normally (but not necessarily) incorporates Pareto optimality.
- A density estimator, which is responsible for maintaining diversity, and therefore, keeping the MOEA from converging to a single solution.

Selection Mechanisms



There are three main types of MOEAs in current use:

- Pareto-based MOEAs
- Decomposition-based MOEAs
- Indicator-based MOEAs

Selection Mechanisms

Pareto-based MOEAs

These are the traditional MOEAs in which the selection mechanism is based on Pareto optimality. Most of them adopt some form of nondominated sorting and a density estimator (e.g., crowding, fitness sharing, entropy, adaptive grids, parallel coordinates, etc.).

Main limitations

Scalability in objective function space is clearly a limitation of Pareto-based MOEAs unless a significantly larger population size is adopted. Another alternative is to change the density estimator, but most people don't seem to be interested in moving in that direction.

Selection Mechanisms

Decomposition-based MOEAs

The core idea of these approaches is to transform a multi-objective problem into several single-objective optimization problems which are simultaneously solved using information from its neighboring subproblems.

Main limitations

The performance of decomposition-based MOEAs relies on the scalarizing function that they adopt. They are also sensitive to the method used to generate weights. However, they are scalable in objective function space (although an increase in the number of objectives will increase the population size).

Selection Mechanisms

Indicator-based MOEAs

The original idea was to adopt a performance indicator for the selection mechanism of a MOEA. However, some researchers discovered that the mere use of a performance indicator in the density estimator was enough to have a good performance (e.g., SMS-EMOA).

Main limitations

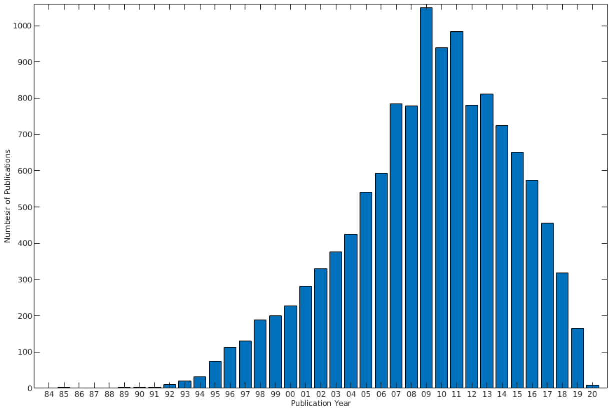
The only performance indicator which is known to be fully Pareto compliant is computationally expensive in high dimensionality (in objective space). Other performance indicators are available, some of which are weakly Pareto compliant (e.g., $R2$ and IGD+). However, researchers don't seem to like them much.

Density Estimators

This is an important component of modern MOEAs which is required to generate several elements of the Pareto optimal set in a single run. The main methods that have been adopted are the following:

- Fitness sharing
- Clustering
- Entropy
- Adaptive grids
- Crowding
- Performance indicators
- Parallel coordinates

Number of papers published per year (early 2020)

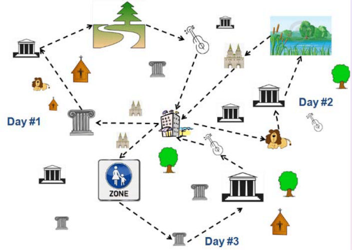


Some Sample Applications

Van der Blom et al. [2019] developed an approach that was able to learn heuristic rules on building spatial design by data-mining multi-objective optimization results. Then, this information was used to gain new insights that can help architects to build better spatial designs with respect to thermal and structural performance.

Koen van der Blom, Sjonnie Boonstra, Hèrm Hofmeyer and Michael Emmerich, "**Analysing Optimisation Data for Multicriteria Building Spatial Design**", in Kalyanmoy Deb et al. (Editors), *Evolutionary Multi-Criterion Optimization, 10th International Conference, EMO 2019*, pp. 671–682, Springer. Lecture Notes in Computer Science Vol. 11411, East Lansing, Michigan, USA, March 10–13, 2019.

Some Sample Applications



Zheng and Liao [2019] used a multi-objective approach based on ant colony optimization and differential evolution for designing personalized tour routes for heterogeneous tourist groups. The authors presented a case study of Kulangsu, which is an islet off the coast of Xiamen, China.

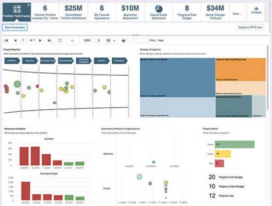
Weimin Zheng and Zhixue Liao, "**Using a Heuristic Approach to Design Personalized Tour Routes for Heterogeneous Tourist Groups**", *Tourism Management*, Vol. 72, pp. 313–325, 2019.

Some Sample Applications

Cabrera et al. [2018] adopted a local search procedure and a multi-objective metaheuristic to optimize the intensity-modulated radiation therapy (IMRT) for cancer treatment. When treating this as a multi-objective problem, the idea is to lead to a set of dose distributions that, depending on both dose prescription and physician preferences, can be selected as the preferred treatment for a patient.

Guillermo Cabrera, Matthias Ehrgott, Andrew J. Mason and Andrea Raith, "**A Metaheuristic Approach to Solve the Multiobjective Beam Angle Optimization Problem in Intensity-Modulated Radiation Therapy**", *International Transactions in Operational Research*, Vol. 25, No. 1, pp. 243–268, January 2018.

Some Sample Applications



Xiao et al. [2018] proposed an extension of MOEA/D which is based on reference distance to solve software project portfolio optimization problems. In this problem, the aim is that a large software company selects their project portfolios to gain maximum return with limited resources under many constraints.

Jing Xiao, Jing-Jing Li, Xi-Xi Hong, Min-Mei Huang, Xiao-Min Hu, Yong Tang and Chang-Qin Huang, "**An Improved MOEA/D Based on Reference Distance for Software Project Portfolio Optimization**", *Complexity*, Article Number: 3051854, 2018.

Research representative of the current trends

- Schemes for solving many-objective problems (i.e., with 4 or more objectives) efficiently and effectively.
- Hybridizations of MOEAs with mathematical programming techniques (e.g., gradient-based methods).
- Use of surrogate and parallel methods to deal with expensive objective functions.
- MOEAs for solving minus versions of well-known test problems (e.g., DTLZ^{-1} and WFG^{-1} test problems).
- Novel applications (e.g., in bioinformatics, medicine, operating systems, etc.).
- Solution of multi-modal multi-objective optimization problems.

Future Challenges

- Can we combine different operators and algorithmic components to produce tailored (and highly competitive) multi-objective evolutionary algorithms?
- Can we design selection mechanisms for multi-objective evolutionary algorithms different from the ones currently available?
- Can we design better density estimators and performance indicators for many-objective optimization problems?
- Specialized schemes (e.g., based on cooperative coevolution) for dealing with large scale MOEAs (e.g., with 5,000 or 10,000 decision variables)
- Multi-objective hyper-heuristics for continuous optimization.

To know more about evolutionary multi-objective optimization

Please visit the new webpage of the EMOO repository:

https://emoo.cs.cinvestav.mx/

To know more about evolutionary multi-objective optimization

The EMOO repository currently contains:

- Over 12,570 bibliographic references including 303 PhD theses, 53 Masters theses, over 6155 journal papers and over 4495 conference papers.
- Contact information of 79 EMOO researchers.
- Public domain implementations of SPEA, NSGA, NSGA-II, the microGA, MOGA, ϵ-MOEA, MOPSO and PAES, among others.